

ADOPTION AND ABANDONMENT OF DUAL EXCHANGE RATE SYSTEMS*

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Abstract:

When facing persistent balance of payments problems, some countries have resorted to a dual exchange rate system as an alternative to a uniform exchange rate adjustment. Typically, under the dual system, certain selected transactions take place at a fixed official exchange rate, while the remaining transactions are effected at a more depreciated rate, which is usually determined by market forces. This paper examines the circumstances that lead to the adoption of a dual system, and the conditions under which the foreign exchange market can be unified successfully at a later stage. In this paper the adoption of the dual system is linked to the unsustainability of a crawling peg (or a fixed exchange rate) system in the presence of large budget deficits. We show that the initial spread between the financial and the commercial exchange rates and the extent of capital flight largely depends on whether the switch in regime is anticipated or unanticipated. Although the dual system improves the external position of the economy, to the extent that there is no change in domestic policies the country will continue to experience a deficit in the balance of payments. A correction of these policies is the only enduring solution to the external imbalance, and a precondition for a successful unification of the foreign exchange market. If the economy unifies the foreign exchange market into a crawling peg, the financial exchange rate could be an adequate indicator of the initial level at which the new exchange rate should be set if a capital outflow is to be avoided. If the economy instead moves to a flexible exchange rate system, the initial value of the exchange rate could be higher or lower than the prevailing financial exchange rate.

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1. Introduction

Dual exchange rate system have been widely used by developing countries. Under these systems, one exchange market is generally reserved for current account transactions, which take place at a commercial (or official) exchange rate that is usually pegged by the central bank. The second market is essentially geared towards capital account transactions, which take place at a financial (or parallel) exchange rate that is freely determined by market forces.

Dual exchange rate systems are usually implemented on a temporary basis. They are generally adopted by countries facing severe balance of payments problems, mostly associated with significant capital outflows. The adoption of a dual system under these circumstances is frequently accompanied by a marked improvement in the balance of payments, since capital outflow pressures no longer erode central bank reserves but lead to a depreciation of the financial exchange rate. This ability of dual system to maintain the external situation under control without the need of a devaluation the official exchange rate or to move to a flexible exchange rate system² in most cases is only transitory. To the extent that the balance of payments problems were the result of expansionary monetary and/or fiscal policies, these problems will sooner or later reappear unless the underlying domestic policies are corrected to make them consistent with equilibrium in the external sector. Once the domestic policies are corrected and confidence is restored there is no compelling reason to maintain a dual system, particularly taking into account the distortions and enforcement problems usually created by these systems and the opposition they face from some international organizations. Therefore, at some point, the authorities have to consider the possibility of abandoning the dual system and adopting some unified exchange rate regime.

Although the working of an economy under a dual exchange rate system has been studied rather thoroughly³, the issues associated with the adoption and the abandonment of dual systems have received considerably less attention. To our knowledge, there is no analytical work that examines the change from a fixed exchange rate system to a dual system; and only some of aspects regarding the unification of the foreign exchange market have been discussed in some detail⁴. The purpose of this paper is to examine the behavior of the economy resulting from both the adoption of a dual system and then the return to a unified system in the context of a relatively simple model.

The rest of the paper is organized as follows. In Section II, we describe the behavior of an economy under a crawling peg system⁵ using a model of the assets approach to the balance of payments⁶. We show that the adoption of expansionary policies may turn the crawling peg system unsustainable, and assume that the authorities decide to switch to a dual exchange rate system. We then modify the relevant equations of the model so as to describe the economy under a dual system⁷. We analyze the behavior of the balance of payments and the exchange rates upon the adoption of the dual system, distinguishing between anticipated and unanticipated changes in regime. We also show that the balance of payments relief obtained by the adoption of the dual system is only transitory unless the underlying domestic policies are corrected. In Section III, we assume that after correcting these policies, the authorities abandon the dual system and adopt, alternatively, a unified crawling peg or a unified floating system. We discuss the behavior of the balance of payments and the exchange rate in each case, again distinguishing between anticipated and unanticipated changes in regime. Finally, in Section IV, we summarize our results and draw some policy implications.

II. Balance of Payments Problems and the Adoption of Dual Exchange Rate Systems

A. The Crawling Peg System

Consider a small economy under a crawling peg system. Let the nominal exchange rate be denoted by e and the rate of crawl by π . Assume that the private sector allocates its wealth between two noninterest bearing assets: domestic money, denoted by M , and foreign money, denoted by f . The nominal stock of private wealth, W , is thus defined by

$$W = M + ef \tag{1}$$

Let the fraction of wealth that is allocated to domestic money be denoted by δ and let δ be a decreasing function of the expected rate of depreciation of the nominal exchange rate, which we assume to be equal to the actual rate of depreciation π . Thus,

$$M = \delta(\pi) W \quad 0 < \delta(\pi) < 1 \quad \delta'(\pi) < 0 \tag{2}$$

Using (2) to replace W in (1), we obtain the portfolio equilibrium condition

$$m = \lambda(\pi) f \quad \lambda(\pi) > 0 \quad \lambda'(\pi) < 0 \tag{3}$$

where $\lambda(\pi) = [\delta(\pi) / (1 - \delta(\pi))]$, and $m = (M/e)$ is the stock of domestic money in terms of foreign currency, which for simplicity will be referred to as the real stock of domestic money. Equation (3) is represented by the ray OP in Figure 1. Since it is assumed that the central bank will sell or buy foreign currency at the predetermined exchange rate, and that portfolio equilibrium is attained instantaneously, the economy is always at a point along OP . In order to describe the adjustment of the economy along this schedule, however, we must also examine the evolution of the stocks of domestic and foreign money (i.e. the stock of total wealth).

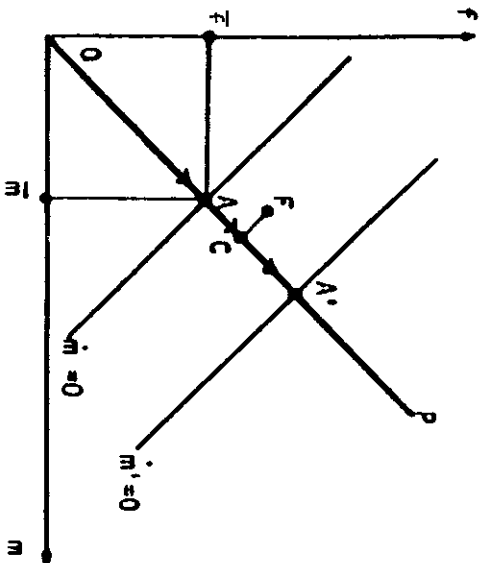


FIGURE 1

The economy produces and consumes traded and nontraded goods. Output of traded goods, y_T , and nontraded goods, y_N , are assumed to be fixed. Private sector nominal expenditure is a fixed proportion, α , of nominal wealth W . A constant fraction α of private sector nominal expenditure is devoted to traded goods, while fraction $(1-\alpha)$ is devoted to nontraded good. Defining the units of traded goods so that their price in terms of foreign money is equal to one, their domestic currency price is equal to the exchange rate, e .⁸ The domestic currency price of nontraded goods, p , adjusts so as to clear the nontraded goods market. Denoting by c_T and c_N the private sector consumption of traded and nontraded goods, we obtain.

$$c_T = \alpha(m + f) \quad (4)$$

$$c_N = (1-\alpha) \alpha (m + f)r \quad (5)$$

where $r = (e/p)$ is the real exchange rate, defined as the relative price of traded with respect to nontraded goods.

Public sector total expenditure, g , and taxes, t , are assumed to be fixed in terms of traded goods. Out of total expenditure g , g_T is devoted to traded goods and g_N to nontraded goods, with

$$g = g_T + g_N \quad (6)$$

Equilibrium in the nontraded goods market requires that private sector demand plus public sector demand be equal to output.

$$[(1-\alpha) \alpha (m+f) + g_N] r = y_N \quad (7)$$

Thus, the real exchange rate r appreciates with increases in private sector wealth and with increases in public sector expenditure in nontraded goods.

It is assumed that domestic credit creation is used to finance the public sector deficit.⁹ Thus

$$\dot{D} = e(g-t) \quad (8)$$

where D is domestic credit, and a dot over a variable denotes its derivative with respect to time. Using R to denote the stock of international reserves expressed in foreign currency, the balance of payments is equal to

$$\dot{R} = [y_T - \alpha \alpha (m+f) - g_T] - \dot{f} \quad (9)$$

where the expression in brackets represents the current account balance, and the other term represents the capital account balance. The change in the nominal stock of domestic money is given by y^{10}

$$\dot{M} = e \dot{R} + \dot{D} = e [y_T - \alpha \alpha (m+f) + g_N - t] - e \dot{f} \quad (10)$$

From equation (3) we obtain

$$\dot{f} = [\lambda(\pi)]^{-1} \dot{m} \quad (11)$$

Using (11) to replace \dot{f} in (10), and noting that $m = (M/e)$, we obtain

$$\dot{m} = \frac{\lambda(\pi)}{1 + \lambda(\pi)} [y_T + \alpha \alpha (m+f) + g_N - t - \pi m] \quad (12)$$

Equation (12) describes the evolution of the real stock of domestic money. Setting $\dot{m} = 0$, we obtain the condition.

$$\alpha \alpha (m+f) = y_T + g_N - t - \pi m \quad (13)$$

Equation (13) is represented by curve $\bar{m} = 0$ in Figure 1. To the left of this curve m increases, and to the right of this curve m declines. Thus, the economy moves along curve OP until it reaches a stationary equilibrium at A , where the stock of foreign money and the real stock of domestic money stay constant.¹¹

Using (3) and (13), the stationary stock of foreign money, \bar{f} , and the stationary real stock of domestic money, \bar{m} , are given by

$$\bar{m} = \frac{\lambda(\pi) [y_T + g_N - t]}{\alpha \alpha [\lambda(\pi) + 1] + \pi \lambda(\pi)} \quad (14)$$

$$\bar{f} = \frac{[y_T + g_N - t]}{\alpha \alpha [\lambda(\pi) + 1] \pi \lambda(\pi)} \quad (15)$$

Since in stationary equilibrium the stock of foreign money is constant and the capital account is in equilibrium, the balance of payments is equal to the current account balance. From equation (9) and (13), it follows that in stationary equilibrium

$$\dot{R} = t - g_T - g_N + \pi m \quad (16)$$

The change in international reserves is equal to the public sector surplus (or deficit) plus the proceeds from the inflation tax.

Under the assumptions of the model, as long as the right and side of (16) is positive or zero, there is no reason for the private sector to doubt that the crawling peg system will be maintained, and thus no balance of payments crisis need occur.¹² However, the adoption of expansionary policies would originate an excess demand for foreign exchange, causing a decline in the international reserves of the central bank, and thereby compromising the sustainability of the crawling peg system.

Assume, for example, that the economy is initially on point A in Figure 1, and that at this point the public sector deficit is exactly equal to the inflation tax, so that the balance of payments is in equilibrium. Now, assume that the public sector reduces taxes t , so that the $\bar{m} = 0$ curve shifts to the right to $\bar{m}' = 0$. The new stationary equilibrium will be A' , and the economy will adjust from A to A' gradually along the OP curve. In the process of adjustment, the real exchange appreciates and the current account turns into deficit due to the increase in real wealth; the capital account also turns into deficit due to

the private sector accumulation of foreign money. Faced with this loss in reserves, and a situation that is unsustainable in the long run, the authorities may decide to switch to a dual exchange rate system by letting the financial transactions to be settled in a free market so as to stop the capital outflow. Furthermore, if the private sector anticipates this change in regime, and expects the financial exchange rate to depreciate with respect to the commercial exchange rate, it will shift its portfolio towards foreign money thereby accelerating the capital outflow and producing a balance of payments crisis that forces the authorities into an immediate adoption of the dual system. A more detailed analysis of this process, however, requires that we first describe the evolution of the various variables under a dual exchange rate system.

B. Dual Exchange Rate System

We assume that under the dual system all financial transactions take place in a free market, and all the commercial transactions take place at the commercial (official) exchange rate (e), which continues crawling at a rate π due to central bank intervention¹³. Since the financial exchange rate adjusts so as to clear the free market, net capital flows are zero, and the stock of foreign money stays constant at the level outstanding at the time the dual system is adopted. Denoting this stock by f_0 , and the financial exchange rate by x , private sector nominal wealth is equal to

$$W = M + x f_0 \tag{17}$$

Thus, private sector consumption of traded and nontraded goods is described by

$$c_T = \alpha a (m + s f_0) \tag{18}$$

$$c_N = (1 - \alpha) a (m + s f_0) r \tag{19}$$

where $s = (x/e)$ indicates the spread between the financial and the commercial exchange rate, and $m = (M/e)$.

Equilibrium in the nontraded goods market now requires

$$[(1 - \alpha) a (m + s f_0) + g_N] r = Y_N \tag{20}$$

The change in international reserves under the dual system is equal to the current account balance since only commercial transactions take place at the commercial exchange rate,

$$\dot{R} = Y_T - \alpha a (m + s f_0) - g_T \tag{21}$$

Since domestic credit creation continues to obey equation (8), the change in the real stock of domestic money follows

$$\dot{m} = Y_T - \alpha a (m + s f_0) + g_N - t - \pi m \tag{22}$$

Setting $\dot{m} = 0$ we obtain the condition

$$\alpha a (m + s f_0) = Y_T + g_N - t - \pi m \tag{23}$$

Equation (23) is represented by curve $\dot{m} = 0$ in Figure 2. To the left of this curve m increases, and to the right m declines.

Since financial transactions take place at the free market, the composition of the private sector portfolio will depend on the expected rate of depreciation of the financial exchange rate. Assuming perfect foresight,

$$m = \lambda (\dot{x}/x) s f_0 = \lambda [(\dot{s}/s) + \pi] s f_0 \tag{24}$$

Equation (24) describes the evolution of the spread between the exchange rates, and can be converted into

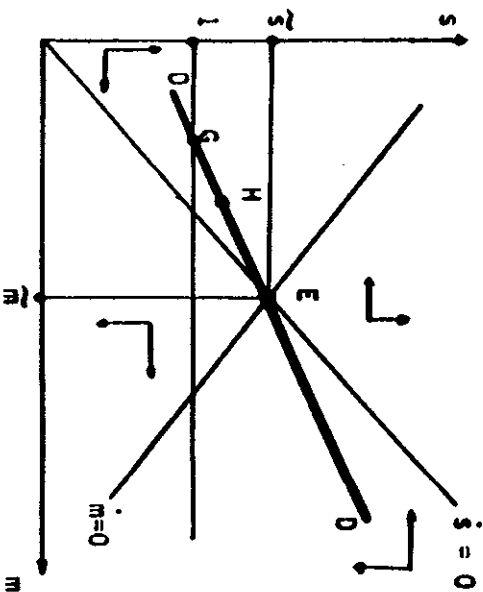
$$(\dot{s}/s) = h (m/s f_0) - \pi \quad h' = (1/\lambda') < 0 \tag{25}$$

Setting $\dot{s} = 0$ we obtain the condition

$$h (m/s f_0) = \pi \tag{26}$$

Equation (26) is represented by the $\dot{s} = 0$ schedule in Figure 2. Above this schedule s increases, and below this curve s declines. The system exhibits saddle point stability. It is assumed that in the absence of anticipated disturbances the variables adjust along the convergent path DD, until they reach a stationary equilibrium at E. The stationary equilibrium values of the spread, \bar{s} , and the real stock of money, \bar{m} , are

FIGURE 2



$$\bar{m} = \frac{\lambda(\pi) [y_T + g_N - 1]}{\alpha [\lambda(\pi) + 1] + \pi \lambda(\pi)} \tag{27}$$

$$\bar{s} = \frac{1}{f_0} \frac{[y_T + g_N - 1]}{\alpha [\lambda(\pi) + 1] + \pi \lambda(\pi)} \tag{28}$$

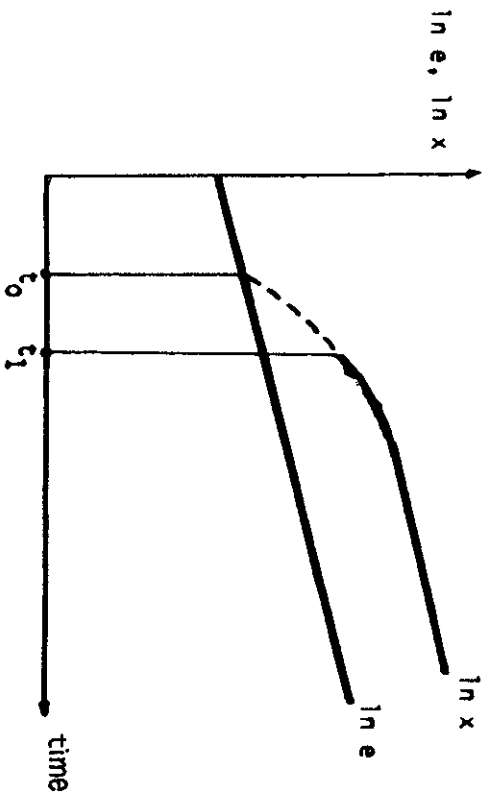
We are now in condition of examining in more detail the transition from the crawling peg system to the dual exchange rate system. As mentioned previously, the adoption of expansionary policies under the crawling peg system produces an appreciation of the real exchange rate, and turns both the current and the capital account of the balance of payments into deficit, as the economy adjusts along a path such as AA' in Figure 1. As the level of international reserves declines, the authorities may decide to switch to a dual exchange rate system in order to stop at least the loss of international reserves arising from capital outflows. The precise consequences of this switch will depend on the decision rule of the authorities and the extent to which the private sector anticipates the change in regime. For example, assume that the authorities decide that they will switch to a dual system when international reserves reach a critical level R_0 . Furthermore, assume that the private sector knows the authorities decision rule and understands the consequences of the change in regime. Under these conditions, the crawling peg system will end with a speculative attack on the international reserves of the central bank, as in similar situations already analyzed by the literature on balance of payments crisis¹⁴. Recall that we are examining cases that result in dual systems with a financial exchange rate (x) above the commercial exchange rate (e). Thus, the private sector knows that the financial exchange rate will depreciate with respect to the commercial exchange rate at the time the crawling peg system collapses. Nevertheless, the anticipation of the private sector implies that the financial exchange rate cannot jump above the commercial exchange rate at the time of the switch in regime, since this would imply an anticipated infinite instantaneous rate of return for holders of foreign money. Instead, the financial exchange rate starts depreciating at a rate faster than π at the time of the switch in regime. This increase in the rate of depreciation causes the private sector to shift its portfolio toward foreign money, thus producing the speculative attack on the international reserves of the central bank that causes the crawling peg system to collapse. This process can be illustrated with the help of Figures 1 and 2. While the economy adjusts from A to A' in Figure 1, international reserves decline steadily until the economy reach a point such as C. At that instant, all of the following takes place. The private sector shifts its portfolio towards foreign money, from C to F, producing a capital outflow that reduces international reserves to the critical level R_0 , thus forcing the authorities to switch to a dual exchange rate system. The magnitude of the speculative attack is such that the level of the real stock of domestic money immediately after the attack is initially, under the dual system, represented by a point such as G in Figure 2. At point G the financial exchange rate is equal to the commercial exchange rate ($s=1$), and hence there is no jump in the financial rate at that time. At G, the financial exchange rate starts depreciating faster than the commercial exchange rate thus validating the change in the composition of the private sector portfolio. From then on, the spread s increases while the economy adjusts along the convergent path DE until it reaches a stationary equilibrium at E.

If the change in regime is unanticipated by the private sector, there is no speculative attack preceding the adoption of the dual exchange rate system. In this case the economy

continues under the crawling peg system past point C in Figure 1 with reserves declining steadily. When they reach the critical level R_0 , the authorities switch to a dual system and the financial exchange rate jumps above the commercial rate and the initial position of the economy under the dual system will be some point such as H in Figure 2. From H, the economy will adjust along the path DD, with the spread between the exchange rates increasing, until the economy reaches a stationary equilibrium at E.

Figure 3 shows the alternative paths for the financial exchange rate. While the solid line describes the evolution of the financial exchange rate when the change in regime is unanticipated, the broken line describe it when the change in regime is anticipated¹⁵.

FIGURE 3



The adoption of the dual system, whether anticipated or unanticipated, immediately stops the capital outflow bringing into equilibrium the capital account of the balance of payments. The adoption of the new system, however, does not stop the worsening of the current account of the balance of payments. As the economy adjust towards point E in Figure 2, with s and m increasing, the current account worsens as indicated by equation (21). Furthermore, the real exchange rate continues appreciating as indicated by equation (20). This implies that while the adoption of a dual system may solve the capital flight problem, other policies are needed under the dual system in order to stop completely the worsening of the external situation of the economy.

The inability of the dual system to eliminate completely the external disequilibrium of the economy can be appreciated by looking at the balance of payments situation once the economy reaches the stationary equilibrium E in Figure 2. Using (27) and (28) to replace m and s in (21) we obtain

$$R = t - g_T g_N + \bar{m} \pi \tag{29}$$

The change in international reserves is equal to the public sector surplus (or deficit) plus the proceeds from the inflation tax. Since the comparison of (14) and (27) reveals that \bar{m}

$= \bar{m}$, equation (29) implies that the balance of payments outcome (in stationary equilibrium) under the dual system is the same that would have resulted (in stationary equilibrium) if the economy had remained under the crawling peg system. Therefore, the adoption of a dual system by itself is only useful as a device to improve temporarily the balance of payments by stopping capital outflows that could rapidly deplete the international reserves of the central bank. This, however, does not solve the basic imbalance between aggregate demand and supply that originated the balance of payments problems in the first place. Other supporting policies are needed in order to make the dual system sustainable in the long run.⁸ Since the effects of these policies under the dual system have already been analyzed in some detail elsewhere¹⁷, we do not examine them in this paper. Instead, we assume that they are implemented so as to restore equilibrium in the balance of payments (e.g. R equal to zero in equation (29)); and we now turn to discuss the abandonment of the dual exchange rate system.

III. Abandonment of Dual Exchange Rate Systems

Once the external situation of the economy under dual exchange markets has been stabilized, in the sense of reaching a stationary state with the balance of payments in equilibrium, it is reasonable for the authorities to consider a return to a unified exchange market system. This section examines the effects of unifying the markets into a crawling peg system, and into a floating system.⁸

A. Unification into a Crawling Peg System

The unification into a crawling peg system is illustrated in Figure 4. Recall that the dynamics of the economy under a crawling peg system are described by the ray OP and the curve $\bar{m} = 0$. The economy is always on the ray OP since this ray represents portfolio equilibrium between domestic and foreign money. The real stock of domestic money increases for points to the left of the $\bar{m} = 0$ curve, and declines for points to the right of the $\bar{m} = 0$ curve. Thus, the economy always moves along the ray OP towards point A, the stationary equilibrium of the economy under the crawling peg system. We assume that the rate of crawl of the unified exchange rate will be equal to the rate of crawl of the commercial rate under the dual system. From equations (14) and (27), it follows that the stationary real stock of domestic money will be the same under both systems. Thus, the inflation tax in stationary state under both systems will be the same, which implies that the balance of payments at the stationary equilibrium under both systems will be the same. In other words, if under the dual system the set of policies was consistent with long run balance of payments equilibrium, the same set of policies will also be consistent with long run balance of payments equilibrium under the crawling peg system. Furthermore, equations (15) and (28), together with (14) and (27), imply that real wealth, and consequently the real exchange rate, will be the same in stationary equilibrium under both systems.

In order to examine the short-run effects of the unification it is necessary to know the position of the economy, in terms of Figure 4, at the time the stationary regime takes place. As mentioned above, the real stock of domestic money at the stationary state is the same under both systems for a given set of policies, including public sector expenditure and taxes, and the rate of crawl of the official exchange rate. Thus $\bar{m} = (\bar{M}/\bar{e})$ under the

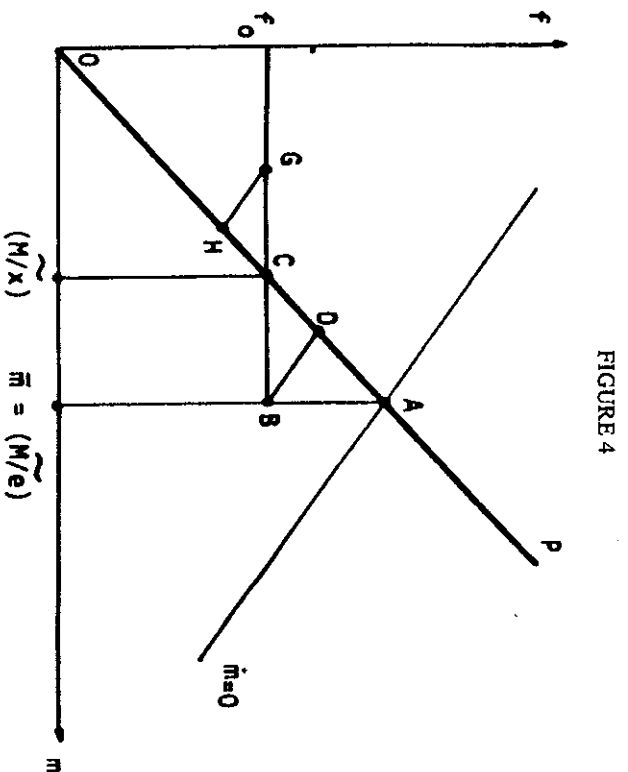


FIGURE 4

dual systems is equal to $\bar{m} = (\bar{M}/\bar{e})$ under the crawling peg system. In addition, since we examine cases in which the free exchange rate is above the official exchange rate under the dual system, $\bar{s} > 1$, equations (15) and (28) imply that f_0 is lower than \bar{f} . In other words, the combination $(\bar{M}/\bar{e}, f_0)$ at the time of unification is represented by a point such as B, exactly below point A. The combination $(\bar{M}/\bar{x}, f_0)$, on the other hand, is represented by point C on the ray OP .⁹

Assume that the exchange market is unified without a maxidevaluation of the official exchange rate. That is, the initial level of the unified rate is equal to the commercial exchange rate under the dual system. In this case, the real stock of domestic money and the stock of foreign money under the crawling peg system at the time of unification are described by point B. Since B is to the right of the ray OP , the private sector is out of portfolio equilibrium, with a share of domestic money in real wealth higher than desired. Thus, the private sector adjusts its portfolio composition, moving immediately from B to D. The increase in private sector holdings of foreign money has a counterpart decline in the international reserves of the central bank, which sells the foreign money at the unified exchange rate. Thus, the unification of the exchange markets without a maxidevaluation of the exchange rate produces an immediate capital outflow. From point D the economy adjusts along ray OP , with a current account surplus that is only partially offset by a capital account deficit, until it reaches point A at which both accounts of the balance of payments remain in equilibrium.

The initial capital outflow that arises when the markets are unified without a maxidevaluation is due to an excess of holdings of domestic money in the portfolio of the private sector. The underlying reason for this excess of domestic money is that the exchange rate relevant for capital transactions is revalued at the time of the unification, since it declines from x under the dual system to e under the unified system. If, alterna-

tively, there is a maxidevaluation that sets the initial level of the unified exchange rate equal to the financial exchange rate of the dual system, the position of the economy at the time of unification would be described by point C on ray OP, and no capital outflow would take place. Thus, a maxidevaluation that sets the initial unified exchange rate equal to the financial exchange rate at the time of unification would eliminate the initial capital outflow. Clearly, a small maxidevaluation would reduce but not eliminate the capital outflow, while a larger maxidevaluation would produce a capital inflow. For example, a maxidevaluation that places the economy at point G at the time unification would produce an initial capital inflow as the private sector sells its excess holdings of foreign money to the central bank in exchange for domestic money in order to move from G to H.

The discussion above assumes that the unification is unanticipated and thus the economy remains at its stationary equilibrium under the dual system until the change in regime takes place. If, alternatively, the unification is anticipated by the private sector, the dynamics of the economy must be reexamined. Assume that the private sector anticipates at time t_0 that the exchange rate will be unified at time t_1 at a rate z and then the unified exchange rate will crawl at a rate π . We know that there cannot be anticipated jumps in the financial exchange rate, and therefore the financial exchange rate at t_1 must be equal to the level at which the exchange rates will be unified z . This implies that the evolution of the various variables depend on the relationship between z and the level that the financial exchange rate would have had at t_1 if the spread at t_0 remained constant until t_1 . Assume that if the spread at t_0 remains constant the financial exchange rate at t_1 is equal to z . Then the economy remains at point E in Figure 5 under the dual system until the unification takes place. At that moment the markets are unified at a rate z , equal to the financial rate under the dual system. As a result, the economy is at point C in Figure 4 under the crawling peg system, there are no initial capital outflows at the time unification, and from C the economy adjusts along OP towards A. Alternatively, assume that if the spread at t_0 remains constant the financial exchange rate at t_1 would be lower than z . This would imply and upward jump in the exchange rate at t_1 . Instead, the financial exchange rate jumps upwards at t_0 moving the economy from E to J in Figure 5. From there, it follows a divergent path until time t_1 , in which it reaches a point such as K where the financial exchange rate under the dual system is equal to z . At t_1 , the exchange markets are unified at the rate z , there is no jump in the exchange rate for capital transactions. The economy under the crawling peg system is initially to the left of point C in Figure 4, and thus an initial capital inflow takes place²⁰. After this, the economy adjust along ray OP towards A. It is now easy to derive the behavior of the economy for the opposite case, in which at the spread prevalent at t_0 the financial exchange rate at t_1 would be higher than z . The financial exchange rate would fall at t_0 moving the economy from E to J' in Figure 5. From there, it would follow a divergent path reaching K' at time t_1 with the financial exchange rate equal to z . At t_1 , the markets would be unified at the rate z , and there would be no jump in the exchange rate for capital transactions. The economy under the crawling peg system would be initially located to the right of point C in Figure 4, and an initial capital outflow would take place after which the economy would adjust along ray OP towards A²¹.

B. Unification into a Floating Exchange Rate System

The foreign exchange markets could alternatively be unified into a floating exchange rate system. In other words, the central bank could stop buying and selling foreign

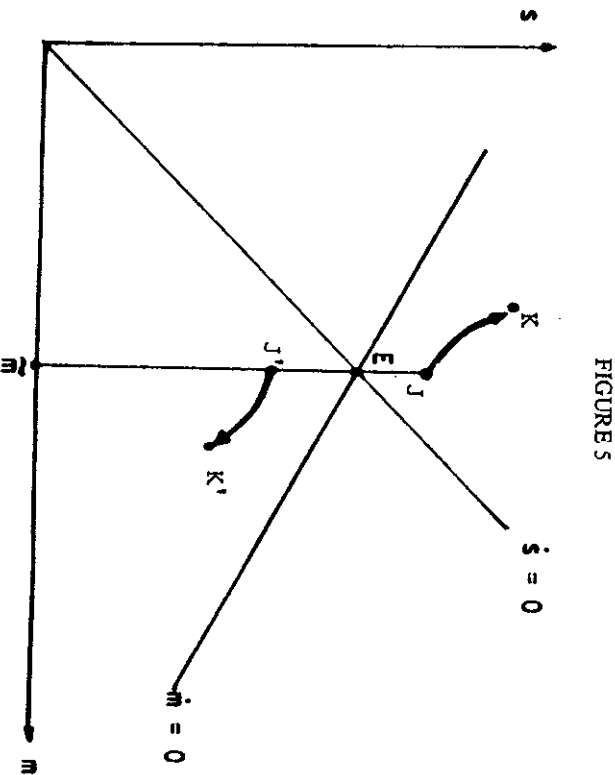


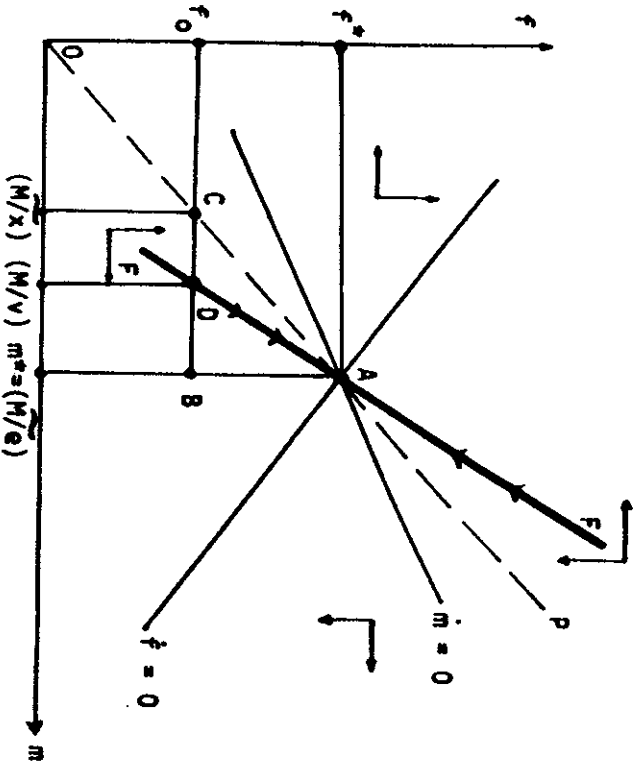
FIGURE 5

exchange for commercial transactions, and thus let all the transactions to be settled in the free market. The analysis of this case requires that we first describe the evolution of the various variables under the floating system. This is done in the appendix.

As we show in the appendix, under a flexible exchange rate system the model exhibits saddle path stability, as illustrated in Figure 6. The economy must always be on that at the stationary equilibrium the real stock of domestic money $m^* = (M/V)$ is equal to \bar{m} , and the budget deficit is entirely financed through the inflation tax.

We proceed now to examine the short run effects of unifying the exchange market into a floating system. In order to do so we need to know the position of the economy, in terms of Figure 6, before the unification of the markets. Assuming that the economy was in a stationary equilibrium under the dual system, the position of the economy can be described by points B and C. Since we are analyzing cases in which under the dual system the financial exchange rate is above the official rate, $\tilde{s} > 1$, equations (28) and (A.12) (from the appendix) imply that $f_0 < f^*$. Recalling that $m^* = \bar{m}$, it follows that the combination $((M/e), f_0)$ is described by point B exactly below A. Point C, on the other hand, describes the combination $(M/X, f_0)^{22}$. Assume now that the exchange market is unified into a floating system. Since the stock of foreign money is equal to f_0 and the economy must be on curve FF under the floating system, at the time of unification the real stock of domestic money adjusts so that the economy moves immediately to point D. Since the nominal stock of domestic money is also given at the time of unification, the variable that adjusts to place the economy in point D is the floating exchange rate v . From Figure 6, since $(M/X) < (M/V) < (M/e)$, it follows that $x > v > e$. In other words, the example in Figure 6 shows that at the time of unification the unified floating exchange rate will be between the commercial and the financial exchange rates of dual system. This result, however,

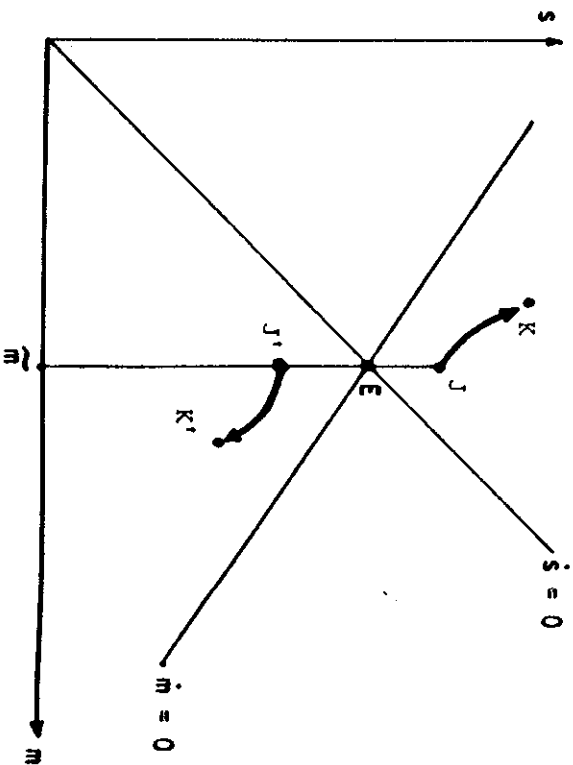
FIGURE 6



does not necessarily hold in all cases. It is possible for the curve FF to be flatter than the ray OP, and thus for point D to be to the left of C. In this case, at the time of unification the floating exchange rate will be higher than both, the commercial and the financial exchange rate of the dual system. What is not possible, is for point D to be to the right of point B. Thus, the floating exchange rate necessarily depreciates with respect to the commercial exchange rate of the dual system. Once in point D the economy adjusts along FF towards A with a capital account deficit, arising from the accumulation of foreign money by the private sector, that is matched by a current account surplus.

The discussion above assumes that the unification was unanticipated, and thus that the economy remained at its stationary equilibrium under the dual system. If, alternatively, the unification is anticipated by the private sector the dynamics of the economy may be different. Assume that the private sector anticipates at t_0 that the exchange market will be unified into a floating system at t_1 . Since there cannot be anticipated jumps in the exchange rate that applies to capital transactions, the evolution of the economy will depend on the position of the curve FF with respect to the ray OP in Figure 6. For example, assume that curve FF coincides with ray OP so that point D coincides with point C. Then, the economy remains in stationary equilibrium under the dual system in point E in Figure 7. At the time of unification $(M/x) = (M/v)$, which implies $x = v$, so there is no jump in the exchange rate that applies to capital transactions. Assume, alternatively, that curve FF is steeper than ray OP, as in the example presented in Figure 6, so that point D is to the right of C. If the economy were to remain at

FIGURE 7



stationary equilibrium under the dual system, at t_1 the exchange rate that applies to capital transactions would fall from x to v . Since the private sector knows that this would be the consequence, the financial exchange rate instead falls immediately at t_0 moving the economy under the dual system from E to J in Figure 7. From there, the economy adjusts along a divergent path with m rising and s declining so that $(M/x) = m/s$ increases. At time t_1 , when the unification takes place, M/x has risen sufficiently to be equal to (M/v) , represented by point D in Figure 6. So, at the time of unification $x = v$, and there is no jump in the exchange rate. From the previous analysis it follows what if the FF curve were flatter than the ray OP, the financial exchange rate would depreciate at t_0 moving the economy under the dual system from E to J in Figure 7. From there, m would decline and s would rise, so that $(M/x) = m/s$ would decline until the unification takes place. At that time, $x=v$ and there would be no anticipated jump in the exchange rate that applies to capital transactions.

IV. Final Remarks

In this section we will briefly discuss some policy aspects regarding the adoption and usefulness of a dual exchange rate system. Our focus will be on the economic circumstances that usually lead countries to adopt dual exchange rates, possible alternatives to its adoption and unification of the market for foreign exchange.

Most countries tend to adopt a dual exchange rate system in the face of significant capital outflows that threaten their holdings of international reserves. The typical si-

nation is similar to the one described in section II of this paper. Under a crawling peg system, an imbalance between domestic savings and the government budget deficit results in a "structural" current account deficit. In the absence of a compensating stream of capital inflows, the central bank experiences a continuous loss of international reserves. The fall in reserves increases the threat of a devaluation, and induces capital outflows making the external position of the country even more fragile.

The adoption of the dual exchange rate system by itself equilibrates the capital account of the balance of payments. When the switch to a dual exchange rate system is anticipated, the end of the crawling peg system will coincide with a speculative attack on the central bank that depletes its international reserves. From that time on, the premium on the financial exchange rate continuously increases until it finally reaches its stationary value. The higher expected rate of return on foreign assets is the primary force behind the speculative attack that takes place. When the switch in regime is unanticipated there is an instantaneous jump in the price of the financial exchange rate as economic agents raise the share of foreign assets in their portfolio. While the adoption of the dual exchange rate system solves the capital flight problem, the current account and balance of payments deficits will persist unless the authorities correct their domestic policies. Regardless of the exchange rate system, the government will have to adjust these policies to achieve external balance.

The two most frequent arguments used against the use of dual exchange rate systems are related to the existence of a difference in the levels of the two exchange rates. The first one² emphasizes the distortionary aspects associated of different prices for what is essentially the same good (foreign currency). In the model presented in this paper the dual exchange rate system is equivalent to a tariff on financial transactions. The second argument hinges on the leakages that usually develop between the two markets. For example, when the financial exchange rate is significantly above the commercial exchange rate economic agents can "arbitrage" by switching transactions from one market to the other through overinvoicing and underinvoicing. In this particular case, importers, by overstating the total value of their purchases abroad, are to buy foreign currency at the (cheaper) commercial exchange rate. At the same time, exporters, by understating the total value of their sales abroad, will sell only a fraction of their total revenues to the central bank at the commercial exchange rate while selling the remaining foreign currency in the financial market. These two mechanisms have a negative impact on the "official" trade balance. The currency that is diverted from the commercial exchange market to the financial market represents a capital outflow, since the private sector increases its stock of foreign assets at the expense of the central bank. The change in central bank reserves will now depend on the reported account balance as opposed to the actual one.

If the monetary authorities, in order to avoid the costs associated with dual systems, choose to remain under a crawling peg system they will have to devalue the exchange rate. Following the devaluation there will be a reduction in real money balances and total private wealth, and thus an improvement in the current account. Moreover, portfolio balance conditions will result in capital inflows (as agents reduce their stock of foreign private assets) and an increase in international reserves. However, once the economy reaches its short run equilibrium, as long as the budget situation is not resolved the private sector will start to accumulate wealth, the balance of payments will deteriorate and the central bank will once again face a loss of reserves, forcing a second devaluation of the exchange rate. In this case a clear pattern emerges in which the central bank repeatedly devalues the exchange rate in order to avoid a balance of payment crisis.

Under both exchange rate arrangements the only enduring solution for the balance of payments problems is a reduction in the budget deficit. The choice between moving to a dual exchange rate system and continuing under a crawling peg will depend on the way in which the authorities evaluate the costs of multiple exchange rates against the costs of repeated maxidevaluations, and on the extent to which they perceive each of these policies to be sustainable during the transition to lower deficits. Under the crawling peg, as the central bank repeatedly makes maxidevaluations, creditability problems could arise causing capital outflows and a collapse in the balance of payments. In this case, the economy can clearly maintain a better grip on its external situation through the adoption of a dual exchange rate system.

The dual exchange rate system, however, is usually adopted as a transitory solution. Due to the problems and distortions associated with its implementation countries eventually unify the foreign exchange market. This paper helped to clarify some issues regarding unification. If the underlying imbalance between aggregate demand and aggregate supply is eliminated before the unification, it was shown that the new long run equilibrium is independent of whether in the unified system the exchange rate is floating or crawling. On the other hand, the short run behavior of the economy will obviously be different. While under a floating system the exchange rate will adjust to eliminate any imbalance between demand and supply of foreign exchange, under a crawling peg system the exchange rate is set by the authorities and any imbalance will be eliminated through variations in international reserves.

This takes us to the role that the financial exchange rate, under the dual system, can play as an indicator of exchange rate developments under a unified system. It was argued in section III that if the economy chooses a flexible exchange rate system the new exchange rate will depreciate with respect to the commercial exchange rate. However, the new rate can appreciate or depreciate with respect to the prevailing financial exchange rate. As a result, it would definitely be misleading to argue that the financial exchange rate can be used as an accurate proxy for the exchange rate that would prevail if the markets were unified into a flexible system. On the other hand, if the economy chooses a crawling peg system, the financial exchange rate may be an adequate indicator of the level at which the unified exchange rate should start to crawl if an initial capital outflow is to be avoided. This, of course, assumes that the underlying causes of balance of payment problems have already been addressed, and that the private sector believes this to be so. Failure of those conditions to hold may result in a new balance of payments crisis.

Notes

- 1 This type of institutional arrangement is certainly not intrinsic to a dual exchange rate system, though it is the most commonly used. In other cases, both exchange rates are allowed to float (such as in Italy during the early 1970s), or both are pegged (such as in Mexico during 1984).
- 2 These arguments are discussed in some detail in Fleming (1971) and (1974), Lanyi (1975), and Kiguel (1985).
- 3 See for example Flood (1978), Aizenman (1983), Cumby (1984), Dornbusch (1986), and Frenkel and Razin (1986).
- 4 In particular the reader is referred to Flood and Marion (1983) and Lizondo (1987b).
- 5 A fixed exchange rate system can be viewed as special case of the crawling peg system, in which the rate of depreciation is equal to zero.
- 6 This model is similar to the ones developed for flexible exchange rates by Kouri (1976), and Calvo and Rodriguez (1977).
- 7 The resulting model resembles those of Flood (1978), Kiguel (1984), and Lizondo (1987a).

- 8 This implies that m can also be interpreted as the real stock of domestic money in terms of traded goods.
- 9 Some of the results presented in this paper could be sensitive to the existence of interest bearing public debt. Recent works by Ize and Ortiz (1987) and van Wijnbergen (1988) dealing with balance of payments crises show, using the budget constraint of the public sector, that government debt can be crucial to understand a balance of payments crisis.
- 10 It is assumed that the central bank does not monetize changes in the domestic currency value of international reserves arising from changes in the exchange rate.
- 11 We define a stationary equilibrium as a point at which the real stocks of domestic and foreign money remain constant for a given set of policies. A stationary equilibrium thus defined is not necessarily a long run equilibrium, since a stationary equilibrium may be consistent with declining international reserves, and therefore unsustainable in the long run.
- 12 The reader is referred to Obstfeld (1986a) for a model in which private sector expectations about a sharp change in government macroeconomic policies in case of a speculative attack, in fact cause such an attack. Thus, in this case there actually is a collapse of an exchange rate regime that would otherwise have been viable.
- 13 Most models in the literature assume that all financial transactions take place in a free market, while all the current transactions take place in an official market. See, for example, Alogoskoufis and Porter (1972), Swoboda (1974), Decluive and Steinhilber (1976), Flood (1978), Marston (1981), Flood and Marston (1982), Aizenman (1983), Cumby (1984), Gardner (1984), Kiguel (1984), Pinto (1986) and Obstfeld (1986b). For models that allow for some current transactions in the free market, see Dornbusch (1976), Blejer (1978), Macedo (1982), Dornbusch *et al.* (1983), Nowak (1984), Dornbusch (1986), and Lizondo (1987a).
- 14 See, for example, Krugman (1979), Flood and Garber (1984), and Obstfeld (1986a).
- 15 Figure 3 shows the logarithms of the exchange rates in the vertical axis, so that a constant slope implies a constant rate of depreciation of the exchange rates.
- 16 Since the stationary equilibrium under the dual system is not sustainable, it can be argued that when switching to a dual system the economy will not adjust along the convergent path DD in Figure 2 since the private sector will anticipate a change in policy. While we directly assume that the economy adjusts along the convergent path DD in order to simplify the presentation, this solution is also consistent with the assumption that the private sector expects the public sector to reduce its expenditure in traded goods in order to solve the balance of payments problem. It can be easily shown that changes in public sector expenditure in traded goods do not affect the stationary equilibrium of the economy, and therefore anticipations of such changes do not affect the dynamics of the economy.
- 17 See for example Aizenman (1983), Cumby (1984) and Dornbusch (1986).
- 18 The unification of dual exchange markets, with a crawling and a floating exchange rate, into a unified crawling and a unified floating system is examined in Lizondo (1987b) under the assumption that some current transactions take place in the free market under the dual system. Flood and Marston (1983) discuss the transition from a dual exchange system, in which both exchange rates are floating, to a unified floating system.
- 19 This follows from equation (3), which defines the ray OP, and equations (27) and (28), which determine (M/x) since $(M/x) = \bar{m}/\bar{s}$.
- 20 Point C in Figure 4 shows $(M/x) = \bar{m}/\bar{s}$, where \bar{m} and \bar{s} correspond to point E in Figure 5. The economy under the crawling peg system is initially to the left of point C in Figure 4 because, in the case, $m < \bar{m}$ and $s < \bar{s}$ in Figure 5 at the time of the unification.
- 21 The economy under the crawling peg system would be initially to the right of point C in Figure 4 because, in this case, $m > \bar{m}$ and $s < \bar{s}$ in Figure 5 at the time of unification.
- 22 This follows from equation (27) and (28), which determine (M/x) , and equations (A.11) and (A.12), which can be used to define the ray OP in Figure 6.
- 23 This argument is most forcefully presented in Adams and Greenwood (1985).

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Appendix

In this appendix we extend the model developed in section II for a flexible exchange rate system. The change in the nominal stock of domestic money under the floating system is equal to the change in domestic credit since international reserves remain constant. Thus,

$$M = v(g - t) \quad (\text{A.1})$$

where v is the floating exchange rate. The change in the real stock of domestic money, $m = M/v$, is therefore equal to

$$\dot{m} = (g - t) - m(\dot{v}/v) \quad (\text{A.2})$$

Portfolio equilibrium now requires

$$m = \lambda(\dot{v}/v) f \quad (\text{A.3})$$

Equation (A.3) can be inverted to obtain

$$\dot{v}/v = h(m/f) \quad h' < 0 \quad (\text{A.4})$$

Using (A.4) to replace \dot{v}/v in (A.2) we obtain

$$\dot{m} = (g - t) - m h(m/f) \quad (\text{A.5})$$

Equation (A.5) describes the evolution of the real stock of domestic money under the floating system. Setting $\dot{m} = 0$ in (A.5), we obtain

$$(g - t) = m h(m/f) \quad (\text{A.6})$$

Equation (A.6) is represented by curve $\dot{m} = 0$ in Figure 6. Below this curve m rises, while above this curve m declines. Notice that curve $\dot{m} = 0$ in Figure 6 could also be positively sloped. However, as long as the elasticity of the demand for money with respect to the expected rate of depreciation of the exchange rate is less than one, we would obtain a saddle path as the one presented in Figure 6. We assume that this condition holds.

The change in private sector holdings of foreign money must be equal to the surplus or deficit of the current account of the balance of payments, since the overall balance of payments must be in equilibrium under the floating system. Thus,

$$\dot{f} = Y_T - \alpha a(m + f) - g_T \quad (\text{A.7})$$

Setting $\dot{f} = 0$ we obtain

$$Y_T - g_T = \alpha a(m + f) \quad (\text{A.8})$$

Equation (A.8) is represented by curve $\dot{f} = 0$ in Figure 6. Above this curve f falls, while below this curve f increases.

Equilibrium in the nontraded good markets requires that private plus public sector expenditure be equal to output. Thus,

$$[(1 - \epsilon) a(m + f) + g_N] r = Y_N \quad (\text{A.9})$$

The system formed by equations (A.5) and (A.7) exhibits saddle-point stability. The economy always moves along the convergent path FF towards the stationary equilibrium A. From equation (A.6), the rate of depreciation in stationary equilibrium must be such that the budget deficit is financed by the inflation tax. Assuming that before the unification the balance of payments was brought into equilibrium, the inflation tax under the dual system $\bar{\pi}$ was equal to the public sector deficit. This means that when the stationary equilibrium is unique, the same pair $(\bar{m}, \bar{\pi})$ would be the stationary state solution for the floating system. In other words, in stationary equilibrium under the floating system the real stock of domestic money, denoted by m^* , will be equal to \bar{m} (the stationary state real stock of domestic money under the dual system), and the rate of depreciation of the floating exchange rate (\dot{v}/v) will be equal to $\bar{\pi}$ (the rate of crawl of the commercial exchange rate under the dual system). Thus, in stationary equilibrium

$$(\dot{v}/v)^* = \bar{\pi} \quad (\text{A.10})$$

$$m^* = \frac{\lambda(\bar{\pi}) [Y_T + g_N - t]}{\alpha a [\lambda(\bar{\pi}) + 1] + \pi \lambda(\bar{\pi})} \quad (\text{A.11})$$

Using (A.10) and (A.11) together with (A.4) we obtain

$$f^* = \frac{[Y_T + g_N - t]}{\alpha a [\lambda(\bar{\pi}) + 1] + \pi \lambda(\bar{\pi})} \quad (\text{A.12})$$

where f^* is the stationary state stock of foreign money under the floating system. Equations (28) and (A.12), together with (27) and (A.11), imply that real wealth, and consequently the real exchange rate, will be the same under stationary equilibrium under both systems.