INTERDEPENDENT RISKS, PREVENTION AND INSURANCE. THE CASE OF THE HEALTH MARKET*

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Abstract

It is usually found in the literature on externalities references to the case of communicable diseases and vaccination. However, neither the nature and importance of the phenomenon have frequently been properly addressed, nor a specific and complete proof of the public good case has been provided.

Within the framework of an uncertainty model, this paper studies the phenomenon of interdependent risks and prevention. In an economy with insurance markets it is proved that, with state dependent utility functions and/or incomplete markets, the case of externality of communicable diseases holds.

I. Introduction

It is usually found in Public Finance books and works regarding externalities, references to the case of communicable illnesses and vaccines. However, neither the nature and importance of the phenomenon have frequently been properly ad-

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dressed, nor a complete and specific proof of the public good case has been pro-
vided. For example, Arrow (1963) and Preston (1975) refer to the case of com-
municable diseases in perfectly certain contexts. If there is no uncertainty about
the likelihood of getting ill (or the effectiveness of a vaccine) when a person
notices that she is going to contract an infectious disease, she faces two alterna-
tives: to be vaccinated or pay those who are going to transmit her the disease to
be immunized. This last possibility recognizes the interdependence of the con-
sumer's decisions and it is the cornerstone of most of the externality proofs in
certainty contexts. However, this case is not very interesting because if she knows
that she is going to get sick, it is not unrealistic to assume that the marginal
utility of the last dollar spent for her immunization will be greater than the mar-
ginal utility of her foregone income. In this case, she gets vaccinated and she is
sure of not getting ill regardless of what other people do. So, even though the
externality case could be made in a certainty context it will be, in my opinion,
artificial. Moreover, a certainty proof looses some features of the problem. For
example, some idiosyncratic aspects of the health insurance market.

Since communicable diseases are a manifestation of risk interdependence the
proper setup to analyze this problem is an uncertainty context. As Culyer (1989)
points out: “There was early recognition (e.g. Weisbrod, 1961) that a direct physical
externality might exist in cases of communicable diseases ... an individual in
choosing or rejecting vaccination may fail to take account of the benefits accruing
externally of a reduced probability of the others contracting disease” (pp. 39).
Although many authors agree on the framework where the problem has to be
considered, they do not agree on the importance of the externality. On the one
hand, Stiglitz (1988, pp. 120) says: “Those who are vaccinated incur some cost
(discomfort, time, risk of getting disease from a bad batch of vaccine). They
receive some private benefit, in reduced likelihood of getting the disease, but a
major part of the benefit is a public good, the reduced incidence of the disease in
the community from which all benefit”. On the other hand, Cornes and Sandler
(1986, pp.115) assert that “by immunizing oneself against an infectious disease,
the individual confers a small benefit on one’s fellows, by slightly reducing the
probability of their becoming infected. At the same time, the benefit to oneself is
particularly great”.

In my opinion much of this disagreement arises from the lack of a specific
proof of the externality case. Hence, the objective of the present paper is to study,
in the framework of an uncertainty model, the phenomenon of interdependent
risks, prevention, and its influence on the probability of getting ill. I am particu-
larly interested in providing the circumstances under which the externality case
of communicable diseases holds, and to have a rough assessment, at least, of the
importance of the externality involved.

The sequence of the paper is as follows: First, I analyze the relationship
between the probability of getting ill, its prevention and interdependent risks. Sec-
ond, I study the consumers’ behavior when they have the chance to take prevent-
tive actions and/or buy market insurance. Then, I examine the social optimum for
each of these circumstances. Finally, some concluding remarks are made.
II. Probability of Getting Ill, Interdependent Risks and Prevention

A person takes preventive actions in order to reduce the likelihood of an event. The probability of an individual of getting ill depends on her genetic predisposition, on the preventive measures she takes and on the (communicable) diseases that other members of the society get or could transmit to her, which, in some cases depends on the preventive actions they have taken. Formally,

\[ f_k \left( H/ s_k; s_1, s_2, \ldots, s_n \right) \]

where \( f_k \) is the density function of the \( k \)th individual over the event \( H \), health status. If we consider that there are two possible realizations of the event, 1 being ill and 0 being healthy, then the likelihood of event 1 is

\[ f_k \left( H=1/ s_k; s_1, s_2, \ldots, s_n \right) = \pi_k \left( s_k; s_1, s_2, \ldots, s_n \right) \]

where \( \pi_k \) is the probability of getting ill of the \( k \)th individual and \( s_k \) is the quantity of prevention done by her. In the health market, when one has to face the eventuality of an illness, it is important to distinguish between communicable and noncommunicable diseases. In order to simplify the exposition (and without lost of generality) I will only refer here to preventive measures taken against contagious diseases (e.g., measles, small pox, cholera) and not to non transmissible diseases (e.g., cancer, arterial hypertension). Furthermore, I will assume that all individuals are identical and that the number of persons is fixed. That, permits expressing this probability as a function of own care \( (s_k) \) and the average \( (s \text{ bar}) \) level of care taken by the other agents

\[ \pi_k \left( s_k; s \text{ bar} \right) \] (1)

The average level of prevention is used here as a measure of the interdependence of risks. In other words, how the probability of illness of other people affects mine. When the influence of \( s \text{ bar} \) over \( \pi_k \) is null, the classical assumption of insurance theory, the probabilities of getting ill are independent.

In the case of certain communicable diseases, prevention may be useful in two ways. First, by directly reducing the probability of a person of getting ill. Second, less directly, by reducing the degree of interdependence of risks which influences the likelihood of contracting a disease of this person and others. Consider the following partial effects

\[ \frac{\partial \pi_k}{\partial s_k} < 0 \] (2)

\[ \frac{\partial \pi_k}{\partial s} < 0 \] (3)
The marginal increase in prevention of the $i$th individual produces a decrease over her probability of becoming ill (2). A marginal increase in the average level of prevention reduces the degree of interdependence of risks and so the probability of getting ill of the $k$th individual (3). In contagious diseases, the direct effect (2) of the vaccination is always greater than the indirect one (3). For example, the vaccine to prevent measles is effective in 95% of the cases. Then, for any likelihood of getting measles is obvious that the direct effect of vaccination is greater than the indirect one. Finally, when $n$ is large the agent $k$ increase in prevention have no effect over the average level of prevention (4). The total effect of a marginal increase in $s_k$ on the probability of getting ill of agent $k$ can be deduced with a total differentiation of (1) with respect to $s_k$.

$$\frac{d\pi_k}{ds_k} = \frac{1}{n}$$

(4)

An increase in $s_k$ generally produces a decrease over the probability of getting ill of the $k$th individual. It is clear that the indirect effect of prevention (see second term of the right member) vanishes when there is a large number of consumers. In that case the total effect of the increase in $s_k$ is equal to the partial or direct effect (first term of right member).

### III. Equilibrium for an Exchange Economy

In this section we are going to analyze which is the optimal quantity of prevention that an individual chooses with and without market insurance.

Assume an exchange economy with a great quantity of consumers in which the law of large numbers holds. In this society there exists one good and its endowment is random. There are two states of nature, $i$ and $h$. A representative consumer $k$, risk avert, faces the event of being ill with probability $\pi_k$ and the event of being healthy with the complementary probability.

Good health, one’s own life or the life of related people are very particular goods because they are essentially unique and irreplaceable. No perfect substitutes exist for these goods in any market. This implies that we cannot use the typical expected utility function where the elemental (or von Neumann and Morgenstern) utility functions are the same independently of the state of nature. Consider the following expected utility function

$$EU_k = \pi_k u_i(x) + (1-\pi_k) u_h(x)$$

In the case of an illness that does not imply cure expenses, or in case the indi-
individual has completely insure his income ($x$ will be equal in both states of nature), the irreplaceable characteristic will be given by

$$u_i(x) < u_h(x)$$

The utility of the same income when ill is less than the utility being healthy.\(^2\)

The relationship among marginal utilities of income in the different state of nature is a matter of disagreement in the literature. Some authors -e.g. Viscusi et al. (1987) - state that the marginal utility of income is greater when one is healthy than ill ("Bad luck: I am rich but I am sick"). Others -e.g. Ellis and McGuire (1990) - sustain that the most reasonable assumption is that the marginal utility of income is greater when ill than healthy ("Bad luck: I am sick and I do not have a nickel"). What it is clear is that the relationship depends on the cross derivatives between the marginal utility of income and the state of nature. Unfortunately, this is a fact that can only be determined empirically. Claiming "neutrality" and easiness in the exposition it will be assumed that

$$u'_i(x) = u'_h(x)$$

The marginal utility of the same income is equal being ill than healthy (for further reference see figure 1). However, it is necessarily to point out that first and second order conditions and comparative statics results could be affected by the relationship between marginal utilities chosen.

### 3.1 Prevention without insurance markets

A representative agent $k$th, risk avert, determines the optimal quantity of prevention through the following exercise

$$\text{Max}_{s_k} \ EU_k = \pi_k [s_k; \overline{s}] u_i (W_k - L_k - s_k) + (1-\pi_k [s_k; \overline{s}]) u_h (W_k-s_k)$$

where $W_k$ is the agent wealth and $L_k$ are cure expenses (the other variables have the same meaning used so far). Then, we have income in each state of nature as an argument of elemental utility functions. From here on and in order to avoid embarrassing notation the arguments of $\pi$ will be omitted and partial derivatives will be denoted by a prime (except in particular cases). The arguments of elemental utility functions $i$ and $h$ will also be avoided unless necessary. The first order condition\(^3\) of the maximization problem is

$$\frac{\partial \ EU_k}{\partial s_k} = \frac{\partial \pi_k}{\partial s_k} (u_i - u_h) - \pi_k u'_i - (1-\pi_k) u'_h = 0$$

reordering

$$\frac{\partial \pi_k}{\partial s_k} (u_i - u_h) = \pi_k u'_i + (1-\pi_k) u'_h = 0 \quad (6)$$
The left term is the marginal private benefit of the last dollar spent in prevention. The marginal benefit is the utility gain coming from the marginal reduction of the agent’s probability of getting ill. The right term is the marginal private cost of the last dollar spent in prevention. In other words, the foregone marginal expected utility of income from the last dollar used in prevention.

3.2 Prevention with insurance markets

Now consider the following situation where consumers have the possibility of taking preventive actions and/or contracting health insurance in the market. The agent can buy an insurance policy which covers $Z_k$ of the medical expenses ($L_k$) which the insurer has to pay in case of getting ill. The contract for this service has two parts: a fixed amount, $d$, and a variable premium, $pZ_k$.

I will make two assumptions about insurance companies. First, there is no incompatibility of incentives between insurance companies and insurers. The rationalization of this phenomenon is the following. The insurance companies compute their premiums as if consumers were not trying to cheat them preventing less. This is due to the irreplaceable character of the good which inhibits the rational consumer from deceiving the insurance company by reducing measures of self-protection. In other terms, the amount of income that a consumer has to receive in order to give up one unit of good health is very large. The consequence of this assumption is that $p$ depends on $s_k$ and $s$ bar in the same way I defined in (2)-(5) for $\eta$. This does not preclude the possibility described by Pauly (1968) where, individuals have few incentives to restrict their consumption to the levels that will prevail in case they have to face the total cost of their consumption. However, this situation is not going to be considered here.

The second assumption is: zero profit condition holds in the insurance market. Firms provide contracts at a variable unit premium which is actuarially fair so that $p=\eta$. But, as risks are interdependent, part of the risk is not diversifiable so in order not to have losses, the insurance company has to collect a fixed payment $d$.

Once we clarify these assumptions let’s consider how we get the optimal consumption of $Z_k$ and $s_k$. For doing this we have to

$$\max_{s_k, s_k} EU = \pi_k (s_k, \tilde{s}) u_i (W_k - L_k + (1-p) Z_k - s_k - d) + [1-\pi_k (s_k, \tilde{s})] u_h (W_k - pZ_k - s_k - d)$$

(7)

Here, again, the argument of elemental utility functions is the agent income in each state of nature.

First order conditions of the maximization problem are

$$\frac{\partial EU_k}{\partial s_k} = \frac{\partial \pi_k}{\partial s_k} (u_i - u_h) + (p'Z_k + 1) [-\pi_k u'_i - (1-\pi_k) u'_h] = 0$$

(8)
reordering (8) and (9) we get
\[
\frac{\partial \pi_k}{\partial s_k} (u_i - u_h) - (pZ_k) [\pi_k u_i + (1-\pi_k) u_h'] = \pi_k u_i' + (1-\pi_k) u_h' \quad (8')
\]
\[
\pi_k u_i' = p [\pi_k u_i' + (1-\pi_k) u_h'] \quad (9')
\]
comparing (8') with (6) it can be seen that the introduction of market insurance affects the optimal quantity of prevention by two means. On the one hand, there is a marginal gain coming from the decrease in the price \(p\) of market insurance due to the marginal increase in \(s_k\). This is the second term of the left side where we have the marginal expected utility of income times the reduction in total variable premiums produced by the last dollar spent in prevention. On the other hand, there is a decrease in the marginal benefit from the last dollar spent in prevention because market insurance reduces the income difference between a situation with and without good health. Finally, the marginal cost of prevention (right side) keeps being the foregone expected marginal utility of income from the last dollar used in prevention.

Consider now equation (9'), it means that the marginal benefit of the last unit of insurance bought (left term) and the marginal cost (right term) of it must be equal in equilibrium. More intuitively, the purpose of insurance is to transfer income between different states of nature. Usually this means to transfer income from a good situation to a bad one. Then, in the left side of (9') we have the expected marginal utility of the last dollar transferred to the ill state and, in the right term, the marginal expected utility of income times the price of an additional unit of insurance (the variable unit premium \(p\)).

Recalling our assumption of zero profits it can be deduced from (9) that when \(L_k = Z_k\), the marginal rate of substitution between states \(h\) and \(i\) is equal to the marginal rate of transformation between those two states of nature. In other terms, given that
\[
\frac{u_h' (\bar{x})}{u_i' (\bar{x})} = 1
\]
for the optimal condition to hold \(Z_k\) must be equal to \(L_k\). Then, the agent choice is to have the mean income in both states of nature. However, this is not enough to completely insure utility because good health is an irreplaceable commodity (for further references see figure 1).

Considering the results reached so far it will be interesting to address the consequences of some of the assumptions done. First, go back to the left side of
equation (8') and see that with a state independent utility function (elemental utility function will be equal) if income is equal for both states of nature the first term of this side will vanish. So, there will be no gain in utility terms coming from the reduction in the agent probability of getting ill. However, as it is demonstrated in Appendix III \( dZ/dp < 0 \). Then, it can be drawn that for an overcharge of the premium \( p > \pi_k \) the quantity of insurance acquired \( (Z_k) \) would be less than the medical expenses in case of illness \( (L_k) \). In this case, the optimal allocation implies that income is greater when one is healthy rather than ill, so utility is greater in the healthy state even with a non state dependent utility function.

Finally, look at the second term of the right side of (8') if variable unit premiums and probabilities of illness are independent (see first assumption on insurance companies) the marginal expected utility gain coming from the reduction in \( p \) is annulled. However, we do not get the usual corner solution because a state dependent utility function assure that some prevention is done. In connection with this remark and related to the assumption over the compatibility of incentives and the replaceability of goods some casual empiricism could confirm the soundness of these hypothesis for this paper. It can be found in practice that incentives plans (premium reductions) for prevention taking exist in some branches of the insurance industry and not in others. Health insurance companies does not have those plans although they could monitor at low cost preventive actions (for example, asking certificates of vaccination, cholesterol level, etc.). In the other hand, car and fire insurance offer premium reductions for prevention. This can be explained because car, offices, etc. are replaceable goods. Then, insurance companies cannot act as if individuals were not trying to deceive them; therefore, they have incentives to give reductions in premiums until marginal expected income and cost of these plans are equated.

IV. Social Optimum

I have already determined how consumers choose their desired levels of prevention in an exchange economy. We should see now if this level of prevention is equal to the socially optimum. The technology I will use consist, in more general cases, in maximizing the sum of all utility functions with respect to the private and public good taking into account that the total demand and supply of goods has to be equalized. In some sense this exercise is trivial for our case because we work with a representative agent; however, it is good enough to make our main point.

4.1 Prevention without insurance markets

The (symmetric) social optimum is obtained by maximizing

\[
\sum_{k=1}^{K} \sum_{\xi=1}^{\xi_k} \left[ \pi_k(s_{\xi}; S) u_i(x_{\xi}^i) + [1 - \pi_k(s_{\xi}; S)] u_h(x_{\xi}^h) \right]
\]
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s.t. \[ \sum_{k=1}^{n} x_k^i = \sum_{k=1}^{n} (W_k - L_k - s_k) \]
\[ \sum_{k=1}^{n} x_k^h = \sum_{k=1}^{n} (W_k - s_k) \]

where \( x_k^i \) and \( x_k^h \) is \( kth \) agent income in the ill state and healthy state respectively.

As we are dealing with a representative agent world this problem can be reduced to

\[
\max_{s_k} EU_k = \pi_k \left[ s_k, s_k \right] u_i (W_k - L_k - s_k) + (1 - \pi_k \left[ s_k, s_k \right]) u_h (W - s_k)
\]

The social planner knows that the average level of prevention is equal to the individual level because all agents are equal.

The first order condition \( 13 \) of the maximization problem is

reordering

The left term is the marginal social benefit of the last dollar spent in prevention and the right term is the marginal social cost of it.

Comparing this solution with the private equilibrium in equation (6) we can see that (see figure 2 for further references), the right term expressions in equations (10) and (6) are similar so marginal social cost (CmgS) and marginal private cost (CmgP) of the last dollar spent in prevention is equal for each level of prevention. The marginal social benefit (BmgS) is greater than the private (BmgP) one for each level of prevention. This is because in the social optimum the indirect effect (2) of prevention is considered and, this has an impact because income in the healthy state is greater than in the ill state \( 14 \) (so \( M^I > M^H \)). Furthermore, marginal benefits are a decreasing function of prevention and marginal costs are an increasing function (check this with second order condition). Then, it can be drawn that the desired level of total prevention is greater in this symmetric social equilibrium \( (s^c) \) than in the private one \( (s^p) \).

4.2 Prevention with insurance markets

The process here is analogous to that presented in the other section. So, the social planner problem is similar to (7) except for the replacement in the probability function of the average level of prevention by \( s_k \) (as we explained earlier there are equal in the representative agent case). The first order conditions \( 15 \) (rearranged) for the social (symmetric Pareto) optimum are

\[
\frac{\partial EU_k}{\partial s_k} = \left[ \frac{\partial \pi_k}{\partial s_k} + \frac{\partial \pi_k}{\partial s} \right] (u_i - u_h) - \left[ \pi_k u_i' + (1 - \pi_k) u_h' \right] = 0
\]
reordering

\[
\left[ \frac{\partial \pi_k}{\partial s_k} + \frac{\partial \pi_k}{\partial s} \right] (u_i - u_h) = \left[ \pi_k u_i' + (1 - \pi_k) u_h' \right]
\]
\[
\left[\frac{\partial \pi_k}{\partial s_k} + \frac{\partial \pi_s}{\partial s}\right] (u_t - u_s) - \left(-\frac{dp}{ds}Z_t\right) [\pi_k u'_k + (1-\pi_k) u'_s] = \pi_k u'_k + (1-\pi_k) u'_s \tag{12}
\]

\[
\pi_k u'_k = p \left[\pi_k u'_k + (1-\pi_k) u'_s\right] \tag{13}
\]

the left term in equation (12) is the marginal social benefit from the last dollar spent in prevention and the right term is the marginal social cost of it.

Comparing this solution with the private equilibrium in conditions (8') and (9') it results that, private and social marginal cost are equal for each level of prevention; but, the marginal social benefit of the last dollar spent in prevention is greater than the private one for each level of prevention. The reason here is, again, that in the social optimum the indirect effect of prevention plays a role. Its impact is spread through two channels. One is the gain in utility coming from the reduction in the probability of getting ill. In this case, is crucial good health being an irreplaceable good so \(u_t(x,\bar{x}) < u_s(x,\bar{x})\). The other, is the marginal expected utility gain coming from the extra reduction in total variable premiums for each level of prevention. As marginal benefits are a decreasing function of \(s\) (check this with second order conditions in Appendix II) and marginal costs are an increasing function of \(s\) the socially optimum level of prevention is greater than the resulting from the private equilibrium.

We have proved that if some reasonable conditions hold an increase in prevention for communicable disease will be Pareto optimum. However, we are in a typical free rider problem: everybody would like the other consumers to raise their level of prevention consumption because of the effect over the average level and consequently on the probability of getting ill. But nobody would consume above the levels drawn from (8) and (9) -or (6)- because, as I defined in (4) and (5), when there is a large amount of consumers individual preventive actions by itself have no effect on reducing the degree of interdependence of risks. This is the root of the incentive to act as a free rider.

Once I deduced that an externality is very likely to arise; the question is: Are the extra benefits from immunization greater than private ones? Being aware of the limitations that the representative agent approach means for making assessment of this kind (specially in what concerns to income distribution), the answer will be NO. Because as regarding immunization the direct effect (1) of vaccination are generally greater than the indirect one's (2).

Also, if we make the reasonable assumption that the marginal externality would increase at an increasing rate when the average level of prevention is low and at a decreasing rate when the average level of prevention rises. Then, considering that \(ds/dW>0\) (see Appendix III) and that in developing economies information gaps (not taken into account along the paper) are a very plausible assumption, we could expect that the marginal externality would be particularly greater in developing countries than in developed economies.
V. Concluding Remarks

This paper proved that if certain conditions hold (incomplete markets and/or utility functions that depend on the state of nature in an economy with insurance markets) an increase in prevention levels, for those communicable diseases where it will be possible to reduce the degree of interdependence of risks, would be Pareto optimum. The increase in welfare could not be achieved because there are no incentives for consumers to prevent above actual levels (given the atomistic effect of their actions over mean levels of prevention).

I also suggested that if the marginal externality increases at an increasing rate when the average level of prevention is low and falls quite fast as the mean level of prevention rises, then the marginal externality would be particularly greater in developing countries than in developed economies.

Finally, in proving the results referred above I have introduced some assumptions and I have pointed out some idiosyncratic aspects of the health insurance market. For example, I have dismissed the incompatibility of incentives problem between insurance companies and insurers by using the irreplaceability condition of the good insured. Also, as dependent risks are insured, a fixed payment is introduced for not violating the zero profit condition. These aspects raise important questions when designing an optimal insurance scheme (private or public) which overcomes the externality problems. However, for a proper analysis of these aspects a formal and structured approach to the firm (the insurance company) is needed. So, I think this could be a very fruitful line of study for further research.
APPENDIX I

For second order sufficient condition for a local maximum to hold
\[
\frac{\partial^2 EU_k}{\partial s_k^2} = \frac{\partial^2 \pi_k}{\partial s_k^2} (u_i - u_h) + 2 \frac{\partial \pi_k}{\partial s_k} (u_i' - u_h') + \pi_{i_i} u_i'' + (1-\pi_{i_i}) u_h'' < 0
\]

if the marginal productivity of prevention is decreasing \((\pi'' > 0)\), health is an irreversible good \((u_i < u_h)\) and the agents are risk avert \((u'' < 0)\) it is a sufficient condition that

\[u_i' \geq u_h'
\]

and it is a necessary condition that

\[-\frac{\partial^2 \pi_k}{\partial s_k^2} (u_i - u_h) - \pi_{i_i} u_i'' - (1-\pi_{i_i}) u_h'' > 2 \frac{\partial \pi_k}{\partial s_k} (u_i' - u_h')\]
APPENDIX II

Consider the system of equations of (8) and (9)
\[\frac{\partial \pi_k}{\partial s_k} (u_i - u_h) + (p'Z_k + 1) [-\pi_k u'_i - (1-\pi_k) u'_h] = 0 = F(s_k, Z_k; w_k, L_k, \pi_k, p) \tag{8''}\]
\[(1-p) \pi u'_i - p (1-\pi) u'_h = 0 = G(s_k, Z_k; w_k, L_k, \pi_k, p) \tag{9''}\]

For second order sufficient conditions for a local maximum to hold it is sufficient that
\[F_s < 0, \quad G_z < 0 \land F_z = G_s = 0\]

Differentiating the system of equations (8'') and (9'') (to avoid embarrassing notation the subindex k is omitted)
\[F_z = \pi'' (u_i - u_h) + 2 \pi' (p'Z + 1) (u'_h - u'_i) - p''Z [\pi u'_i + (1-\pi) u'_h] + + (p'Z+1)^2 [\pi u'' + (1-\pi)u'']\]
\[F_z = G_z = \pi' [(1-p) u'_i + p u'_h] - p' [\pi u'_i + (1-\pi) u'_h] + + (p'Z+1) [-\pi (1-p)u'' + (1-\pi) pu'']\]
\[G_z = (1-p)^2 \pi u'' + p^2 (1-\pi)u''\]

Taking into account that the marginal productivity of prevention is decreasing ($\pi'' > 0$), health is an irreplaceable good ($u_i < u_h$) and the agents are risk averts. Also consider that, when $\pi = p$ then $u'_i = u'_h$ and that from equation (8) can be drawn ($p'Z+1 > 0$). Then, we can deduce that $F_s < 0$. In the case where in equilibrium $p > \pi$ then $u'_i > u'_h$; so, we have to analyze the parameter values to reach a conclusion about $F_s$.

In addition, when $\pi = p$ the marginal utilities are equal in both states of nature and given that in the state of nature $i$ the income chosen is equal to that in $h$, $u_i'' = u_h''$. From all these relationships we can infer that $G_s = G_z = 0$. Finally, if agents are risk avert $G_z < 0$. 

APPENDIX III

In this appendix we want to study the following comparative statistic results: \( \frac{dZ}{dW}, \frac{ds}{dW}, \frac{dZ}{dp} \) and \( \frac{ds}{dp} \). In order to reach them, we have to replace \( Z^* \) and \( s^* \) in (8") and (9"), then take derivatives (in a neighborhood of the equilibrium points) respect to \( Z, s, W \) and \( p \) and finally using Cramer rule we get:

\[
\begin{align*}
\frac{ds}{dw} & = \frac{-F_w F_z - G_w G_z}{G_s F_z - F_s G_z} = -\frac{F_w G_z + F_z G_w}{\Delta} \\
\frac{dZ}{dw} & = \frac{F_s - F_w}{G_s G_z} = -\frac{F_s G_z + F_z G_s}{\Delta}
\end{align*}
\]

\[
\begin{align*}
\frac{ds}{dp} & = \frac{-F_p F_z - G_p G_z}{G_s F_z - F_s G_z} = -\frac{F_p G_z + F_z G_p}{\Delta} \\
\frac{dZ}{dp} & = \frac{F_s - F_p}{G_s G_z} = -\frac{F_s G_z + F_z G_s}{\Delta}
\end{align*}
\]

where

\[
\begin{align*}
F_w & = \pi' (u'_i - u'^*_i) - (p'Z+1) [\pi u''_i + (1-\pi) u''_*] \\
G_w & = (1-p) \pi u''_* - p (1-\pi) u''_i \\
F_p & = \pi'Z(u'_i - u'^*_i) - p''Z[\pi u''_i + (1-\pi) u''_*] + Z(p'Z+1) [\pi u''_* + (1-\pi) u''_i] \\
G_p & = -\pi u'_i - (1-\pi) u''_* - (1-p) \pi Z u''_* + p (1-\pi) Z u''_i
\end{align*}
\]

If we analyze the situation where \( p=\pi \) and considering that in this case we have: \( u' > 0, u'' < 0, u'_i = u'^*_i, u''_* = u''_* \) and \( (p'Z+1) > 0 \). So, it can be drawn that \( F_w > 0, G_w = 0, F_p < 0 \) and \( G_p < 0 \). Properly replacing and remembering from appendix II that the divisor is positive we get \( \frac{ds}{dW} > 0, \frac{dZ}{dW} = 0, \frac{ds}{dp} < 0 \) and \( \frac{dZ}{dp} < 0 \).
FIGURE 1

STATE DEPENDANT UTILITY FUNCTION

FIGURE 2

OPTIMAL PREVENTION QUANTITY
Notes

1. Market insurance and self-insurance transfer income between different (random) states of nature. See Ehrlich and Becker (1972).

2. The elemental utility function could have also been expressed as \( u(x, H) \). Where \( u(x,1) < u(x,0) \).

3. Unless I also assume that \( u'_e \geq u'_p \) the agent being risk avert is not enough for the second order sufficient condition for a local maximum to hold. See Appendix I.

4. Although there are masochists, suicidal types, etc. individuals in average are rational.

5. In case of death it will tend to infinity.

6. They do not consume more because they prevent less. In other words, they move along the demand curve, they do not shift it to the east. See Berlinski (1995).

7. Second order sufficient conditions for a local maximum are analyzed in Appendix II.

8. It is important not to forget that the optimal allocation chosen depends on the assumption made about the marginal utility of income. In case \( u_e' > u_p' \) for the optimal condition to hold \( L_e < Z_e \) and if \( u_e' < u_p' \) the first order condition will be fulfilled when \( L_e > Z_e \).

9. With elemental utility functions independent of the state of nature, agent insures income and utility completely when premiums are actuarially fair.


11. This peculiarity has been suggested to me by Alfredo Canavese.

12. I am not considering collection cars and other bizarre cases.

13. Second order sufficient condition for a local maximum is equal to the one in Appendix I except that the derivatives of \( \pi_e \) respect to \( s_e \) are total instead of partial.

14. In this case state dependent utility functions only widen marginal benefits respect to the independent utility function case.

15. Second order sufficient conditions for a local maximum are equal to those presented in Appendix II with the exception that the derivatives of \( \pi_e \) and \( p \) respect to \( s_e \) are total instead of partial.

16. As was established in the other section when elemental utility functions are independent of the state of nature is sufficient that insurance market were incomplete so \( x_e' < x_p' \).

References


